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ASYMPTOTIC PROPERTIES OF THE METHOD OF EMPIRICAL MEAN FOR STATIONARY RANDOM PROCESSES AND HOMOGENEOUS RANDOM FIELDS

The article considers the quality of empirical estimates of unknown parameters of stationary random processes and homogeneous random fields for which the conditions of ergodicity or strong mixing are satisfied. A series of statements on consistency of estimates, asymptotic distribution and large deviations for estimations of unknown parameter obtained by the method of empirical means for independent or weakly dependent observations was formulated.

Keywords: *method of empirical mean, asymptotic properties, consistency, estimate, large deviations, criterion function.*

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АСИМПТОТИЧНІ ВЛАСТИВОСТІ МЕТОДУ ЕМПІРИЧНИХ СЕРЕДНІХ ДЛЯ СТАЦІОНАРНИХ ВИПАДКОВИХ ПРОЦЕСІВ ТА ОДНОРІДНИХ ВИПАДКОВИХ ПОЛІВ

Одним з підходів, що використовується в системному аналізі при прийнятті оптимальних рішень за умов ризику та невизначеності, є стохастичне програмування, яке дозволяє враховувати ймовірнісний характер досліджуваних процесів.

При розв'язанні задач стохастичного програмування не завжди можливо знайти точний екстремум математичного сподівання деяких випадкових функцій, що робить актуальним метод емпіричних середніх, який є одним з підходів, що дозволяє розв'язати вказану проблему і полягає в заміні оціночної функції її емпіричною оцінкою, для якої розв'язується наближена оптимізаційна задача. При цьому умови збіжності істотно залежать від оціночної функції, ймовірнісних властивостей спостережень випадкових об'єктів, метрики просторів, для яких досліджується збіжність, апріорних обмежень на невідомі параметри тощо. В термінології теорії оптимальних рішень ці питання тісно пов'язані з асимптотичними властивостями оцінок невідомих параметрів: консистентністю, асимптотичним розподілом, швидкістю збіжності оцінок тощо.

Чимало задач математичної статистики (оцінювання невідомого параметра за критеріями найменшого квадратичного тощо) можуть бути сформульовані як спеціальні задачі стохастичного програмування зі специфічними обмеженнями на невідомий параметр, підкреслюючи тісний зв'язок між стохастичним програмуванням та методами теорії оцінювання.

У статті розглядається якість емпіричної оцінки невідомих параметрів стаціонарних випадкових процесів та однорідних випадкових полів, для яких виконуються умови ергодичності або строго перемішування. Сформульований ряд тверджень про консистентність, асимптотичний розподіл та великі відхилення оцінок невідомого параметра, отриманих методом емпіричних середніх для незалежних та слабко залежних спостережень.

Ключові слова: *метод емпіричних середніх, асимптотичні властивості, консистентність, оцінка, великі відхилення, оціночна функція.*

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АСИМПТОТИЧЕСКИЕ СВОЙСТВА МЕТОДА ЭМПИРИЧЕСКИХ СРЕДНИХ ДЛЯ СТАЦИОНАРНЫХ СЛУЧАЙНЫХ ПРОЦЕССОВ И ОДНОРОДНЫХ СЛУЧАЙНЫХ ПОЛЕЙ

В статье рассматривается качество эмпирических оценок неизвестных параметров стационарных случайных процессов и однородных случайных полей, для которых выполняются условия эргодичности или сильного перемешивания. Сформулирована серия утверждений о непротиворечивости оценок, асимптотическом распределении и больших уклонениях для оценок неизвестного параметра, полученных методом эмпирических средних для независимых или слабо зависимых наблюдений.

Ключевые слова: *метод эмпирических средних, асимптотические свойства, состоятельность, оценка, большие отклонения, оценочная функция.*

1. Introduction

In article will be considered the next problems:

1. The method of empirical means for discrete and continuous models with dependent observations.
2. The method of empirical means applied to the non-stationary models.
3. The method of empirical means under the restrictions of unknown parameters, described in the form of equalities and inequalities.
4. The method of empirical means for the models, in which the random functions depend on several variables or random fields.
5. The problems of the large deviations for the method of the empirical means.

The general problem of stochastic programming is formulated in [7]. The method of empirical mean is one of the most effective methods of stochastic approximation. The basis of this method is the ideas of the Theory of Estimation, developed by Le Cam [8], Pfanzangl [9] and others. Different aspects of the convergence of this method are considered in [10] and [11].

2. Main Part

Let $\left(Y,\mathcal{L}(Y)\right)$ be some metric space, where $\mathcal{L}(Y)$ is the minimal σ–algebra on Y, and denote by $\|\cdot\|$ the norm in Y.

Let $\{\xi_i, i\in N\}$ be independent identically distributed observations of a random variable defined on a probability space $(\Omega,\Im,P)$ with values in $\left(Y,\mathcal{L}(Y)\right)$, and let ξ be a random variable with the same distribution and taking values in the same metric space. We assume that I is a closed subset in $\Re^l, l\geq 1$ and $f: I\times Y\to\Re$ is a nonnegative function satisfying the following conditions:

1) $f(\mathbf{u},z), \mathbf{u}\in I$, is continuous for all $z\in Y$;
2) for any $\mathbf{u}\in I$, the mapping $f(\mathbf{u},z), z\in Y$, is $\mathcal{L}(Y)$–measurable.

The problem consists in finding the minimum point of the function

$$F(\mathbf{u}) = \mathrm{E}\left(f(\mathbf{u},\xi)\right), \mathbf{u}\in I,$$

and its minimal value.

This problem is approximated by the following one: find the minimum points of the function

$$F_n(\mathbf{u}) = \frac{1}{n}\sum_{i=1}^{n} f(\mathbf{u},\xi_i),$$

and its minimal value.

We give some examples of regression models, which are widely known to specialists in the field of theoretical and applied statistics.

1. $y_i = \sum_{t=1}^{p} x_{it}\alpha_t^0 + \varepsilon_i, \quad i = 1,...,n$.

Here $\varepsilon_i, i = 1,...,n$ are independent or stationary dependent random variables, $\mathbf{x}_i = \{x_{it}, i = \overline{1,p}\}, i = 1,...,n$ are independent identically distributed random vectors, independent of $\varepsilon_i, i = 1,...,n$.

The vector $\boldsymbol{\alpha}^0 = (\alpha_1^0,...,\alpha_p^0)$ is unknown and would be estimated.

2. $y_i = g(\mathbf{x}(i), \boldsymbol{\alpha}^0) + \varepsilon_i, \mathbf{x}(i) \in \Re^p$, where the p-dimensional vector $\mathbf{x}(i)$ and ε_i are mutually independent, and each of the sequences $\{\mathbf{x}(i)\}$ and $\{\varepsilon\}, i = 1,...,n$ is the sequence of independent or stationary random vectors or variables.

Some cost functions characterizing the accuracy of the estimate:

1. $F_n(\alpha) = \frac{1}{n} \sum_{i=1}^{n} \left[y_i - \sum_{j=1}^{p} x_{it}\alpha_t \right]^2$;

2. $F_n(\alpha) = \frac{1}{n} \sum_{t=1}^{n} \left[y_i - g(\mathbf{x}(i), \boldsymbol{\alpha}) \right]^2$;

3. $F_n(\alpha) = \frac{1}{n} \sum_{i=1}^{n} | y_i - \sum_{t=1}^{p} x_{it}\alpha_t |$;

4. $F_n(\alpha) = \frac{1}{n} \sum_{i=1}^{n} | y_i - g(\mathbf{x}(i), \boldsymbol{\alpha}) |$.

Theorem 2.1. [1] *Let the following conditions be satisfied:*

1) *for any* $c > 0$, $E\left(\max_{\|\mathbf{u}\| \le c} f(\mathbf{u}, \xi) \right) < \infty$, *where* $\|\cdot\|$ *is a norm in* $\Re^l, l \ge 1$;

2) *if* $P\{\xi \in Y'\} = 1$, *then for all* $z \in Y'$ *we have* $f(\mathbf{u}, z) \to \infty$ *as* $\|z\| \to \infty$;

3) *there is a unique point* $\mathbf{u}_0$, *at which the function* $F(\mathbf{u})$ *attains its minimum.*

Then, for any n *and* $\omega \in \Omega'$, $P(\Omega') = 1$, *there is at least one vector* $\mathbf{u}_n = \mathbf{u}_n(\omega) \in I$ *for which the minimum value of* $F_n(\mathbf{u})$ *is attained and, for any* $n \ge 1$, *the vector* $\mathbf{u}_n$ *can be chosen to be* G_n'-*measurable, where* $G_n' = G_n \cap \Omega'$ *and* $G_n = \sigma\{\xi_i, i = \overline{1,n}\}$. *In this case, with probability 1,* $\mathbf{u}_n \to \mathbf{u}_0$ *and* $F(\mathbf{u}_n) \to F(\mathbf{u}_0)$.

Theorem 2.2. [1] *Let* $(\Omega, \mathcal{U}, P)$ *be a probability space,* K *is a compact subset of some Banach space with a norm* $\|\cdot\|$. *Suppose that*

$$\{Q_n(s) = Q_n(s,\omega) \in K \times \Omega, n \ge 1\}$$

is a family of real functions satisfying the following conditions:

1) *for any* n, s *the function* $Q_n(s,\omega), \omega \in \Omega$ *is measurable;*

2) *for fixed* n *and* ω *the function* $Q_n(s,\omega), s \in K$ *is continuous;*

3) *for some element* $s' \in K$ *for each* $0 < \gamma < \gamma_0$ *one has*

$$P\left\{ \lim_{T \to \infty} Q_n(s,\omega) = \Phi(s, s_0) \right\} = 1,$$

where $\Phi(s, s_0) > \Phi(s_0, s_0), s \ne s_0$;

4) there exist $\gamma_0 > 0$ and a function $c(\gamma), \gamma > 0$, $c(\gamma) \to 0, \gamma \to 0$, such that for any element $s' \in K$ and any $0 < \gamma < \gamma_0$ one has

$$P\left\{ \overline{\lim_{T\to\infty}} \sup_{\|s-s'\|<\gamma} \left| Q_n(s) - Q_n(s') \right| < c(\gamma) \right\} = 1 .$$

For each $n > 1$ and $\omega \in \Omega$ define an element $s_n = s_n(\omega) \in K$ by the relation

$$Q_n(s_n) = \min_{s \in K} Q_n(s) .$$

If there is more than one point of minimum of the function Q_n we will take any point s_n. Then

$$P\{\|s_n - s_0\| \to 0, Q_n(s_n) \to \Phi(s_0, s_0), n \to \infty\} = 1 .$$

Applying the ergodic theorem, it is possible to show that the claim of Theorem 1 holds true, and in this case the random sequence $\{\xi_i, i \in N\}$ is ergodic and stationary in the restricted sense.

Theorem 2.3. [1] *Let $\{\xi(t), t \in \Re\}$ be a random ergodic process stationary in the restricted sense and defined on the probability space $(\Omega, \Im, P)$ with values in $\Re$. Let the following conditions be satisfied:*

1) *for any $c > 0$, $E\left\{ \max_{\|\mathbf{u}\| \le c} f(\mathbf{u}, \xi(0)) \right\} < \infty$;*

2) *if I is an unbounded set for any $z \in Y'$, and $P\{\xi(t) \in Y' \; \forall t \ge 0\} = 1$, then $f(\mathbf{u}, z) \to \infty$ as $\|\mathbf{u}\| \to \infty$;*

3) *there is a unique element $\mathbf{u}_0 \in I$ for which the minimal value of the function $F(\mathbf{u}) = Ef(\mathbf{u}, \xi(0))$ is attained.*

Then, for all $T > 0$ and $\omega \in \Omega'$, $P(\Omega') = 1$, there is at least one vector $\mathbf{u}(T) \in I$ for which the minimal value of the function

$$F_T(\mathbf{u}) = \frac{1}{T} \int_0^T f(\mathbf{u}, \xi(t)) dt$$

is attained and measurable.

Let $\mathbf{u}_0 = \arg\min F(\mathbf{u})$, where $F(\mathbf{u}) = Ef(\mathbf{u}, \xi(0))$. Then we have

$$P\left\{ \lim_{T\to\infty} \mathbf{u}(T) = \mathbf{u} \right\} = 1, \quad P\left\{ \lim_{T\to\infty} F_T(\mathbf{u}_T) = F(\mathbf{u}_0) \right\} = 1 .$$

Now we consider the case of a non-stationary model. We will suppose that the criterion function depends also on the temporal parameter, i.e., it is a function of three variables. For example, in the discrete time the criterion function has the form

$$F_n(u) = \frac{1}{n} \sum_{i=1}^{n} f(i, u, \xi_i) .$$

As an example, one can take

$$F_n(u) = \frac{1}{n} \sum_{i=1}^{n} \left[y_i - g(i, u) \right]^2$$

or

$$F_n(u) = \frac{1}{n} \sum_{i=1}^{n} | y_i - g(i, u) |$$

for the model of the observation

$$y_i = g(i,u_0) + \xi_i .$$

Such models were considered in [2].

Let's consider one more model connected with problems of nonparametric estimation. We assume also that the unknown parameter is an element of some functional space. For example, one we can consider the problem of the estimation of the unknown function $u(t) \in K$, where K is the compact set of functions defined on [0,1], by observations

$$y_i = u(\frac{j}{n}) + \xi_i, i = 1..., n$$

with some criterion function.

Theorem 2.4. [3] *Suppose that for an ergodic homogeneous in the wide sense random field* $\xi(t) \in Y, t \in \Re^n$ *the conditions below are fulfilled:*

1) *for any* $c > 0$

$$E\left\{ \max_{\|u\|<c} \left(f(\text{u},\xi(0))\right)^2 \right\} < \infty ;$$

2) *if* $\Im$ *is unbounded then for each* $z \in Y$

$$f(\text{u},z) \to \infty, \|u\| \to \infty ;$$

3) *there is a unique element* $u^* \in \Im$ *for which the minimal value of the function* $F(u) = \text{E}f(u,\xi(0))$ *is attained.*

Then for all $T > 0$ *and* $\omega \in \Omega', P(\Omega') = 1$, *there is at least one vector* $u(T) \in \Im$ *for which the minimal value of the function* $F_T(u)$ *and*

$$P\left\{u(T) \to u_0, F_T(u(T)) \to F(u_0), T \to \infty\right\} = 1 .$$

Let $\left\{\xi(\vec{t}) = \xi(\vec{t},\omega), \vec{t} \in \Re^m\right\}, m \geq 0$ be a homogeneous in a strict sense random field on a complete probabilistic space $(\Omega, \mathcal{G}, P)$ with values in some metric space $(Y, \mathcal{B}(Y))$. Suppose that realizations of $\xi(\vec{t})$ are continuous on $\Re^m$ with probability 1. We have a continuous nonnegative function $f : J \times Y \to \Re$, where J is a closed subset of $\Re^l$, $l \geq 1$.

We have the observations $\left\{\xi(\vec{t}) : \|\vec{t}\| < T\right\}, T > 0$. The problem is to find minimum points and the minimal value of the function

$$F(\vec{u}) = Ef\left(\vec{u}, \xi(\vec{0})\right), \vec{u} \in J . \qquad (1)$$

We will investigate the problem (1). This problem is approximated by minimization of the function

$$F_T(\vec{u}) = \frac{\int_{\|t\|<T} b\left(\vec{u}, \xi(\vec{t})\right) dt}{\int_{\|t\|<T} dt}, \vec{u} \in J .$$

Denote

$$b_1(\vec{t}) = b_1(\vec{t}, c) =$$

$$= \frac{E\left(\inf\limits_{\|\vec{u}\|>c} f(\vec{u},\xi(\vec{t})) - Ef\inf\limits_{\|\vec{u}\|>c} f(\vec{u},\xi(\vec{0}))\right)\left(\inf\limits_{\|\vec{u}\|>c} f(\vec{u},\xi(\vec{0})) - Ef\inf\limits_{\|\vec{u}\|>c} f(\vec{u},\xi(\vec{0}))\right)}{E\left(\inf\limits_{\|\vec{u}\|>c} f(\vec{u},\xi(\vec{0})) - E\inf\limits_{\|\vec{u}\|>c} f(\vec{u},\xi(\vec{0}))\right)^2};$$

$$b_2(\vec{t}) = b_2(\vec{t},\vec{u}) = \frac{E\left(f(\vec{u},\xi(\vec{t})) - F(\vec{u})\right)\left(f(\vec{u},\xi(\vec{0})) - F(\vec{u})\right)}{E\left(f(\vec{u},\xi(\vec{0})) - F(\vec{u})\right)^2};$$

$$b_3(\vec{t}) = b_3(\vec{t}, K, \gamma) =$$

$$= \frac{E\left(\Psi(K,\gamma,\xi(\vec{t})) - E\Psi(K,\gamma,\xi(\vec{0}))\right)\left(\Psi(K,\gamma,\xi(\vec{0})) - E\Psi(K,\gamma,\xi(\vec{0}))\right)}{E\left(\Psi(K,\gamma,\xi(\vec{0})) - E\Psi(K,\gamma,\xi(\vec{0}))\right)^2};$$

where

$$\Psi(K,\gamma,z) = \sup_{\vec{u},\vec{v}\in K:\|\vec{u}-\vec{v}\|<\gamma} \left\| f\left(\vec{u},z\right) - f\left(\vec{v},z\right)\right\|.$$

The next theorem takes place.

Theorem 2.5. [3]. *Let the next conditions be fulfilled:*

1) for any $c > 0$

$$E\left\{\max_{\|\vec{u}\|<c}\left(f(\vec{u},\xi(\vec{0}))\right)^2\right\} < \infty;$$

2) J *is unbounded then for each* $z \in Y$

$$f(\vec{u},z) \to \infty, \|\vec{u}\| \to \infty;$$

3) the function (1) *has a unique minimum point* $\vec{u}^*$;

4) for all $c > 0, \vec{u} \in J$ *c > 0,* $\vec{u}$ *∈ J, and any compact K ⊂ J,γ > 0*

$$\int_0^1 \frac{(\ln\rho)^2}{\rho^m}\left(\int_0^\rho \frac{1}{\tau^2}|B_i(\tau)|\,d\tau\right)d\rho < \infty, i = \overline{1,3},$$

where

$$B_i(\tau) = \int_{\|\vec{t}\|<\tau} b_i(\vec{t})d\vec{t}, i = \overline{1,3}.$$

Then for all $T > 0, \omega \in \Omega', P(\Omega') = 1$ *it exists at least one minimum point* $\vec{u}(T) = \vec{u}(T,\omega)$ *of the function* $F_T\left(\vec{u}\right)$. *For any* $T > 0$ *the function* $\vec{u}(T,\omega)$ *can be chosen* $\mathcal{G}'$*-measurable, where* $\mathcal{G}' = \left\{A \in \mathrm{G} : A \subset \Omega'\right\}$.

For any minimum point $\vec{u}(T)$

$$P\left\{\vec{u}(T) \to \vec{u}_0, F_T\left(\vec{u}(T)\right) \to F\left(\vec{u}_0\right), T \to \infty\right\} = 1.$$

Another important property of estimates is their limit distributions. It is important to know that if the true value is an interior point of the domain of admissible values, or it belongs to the boundary of this domain. We will not formulate all the conditions under which one can prove the statement on the limit distribution of the estimate, because these conditions are indeed very complicated. These conditions are given in a full look in [4].

For example, if we have the observations of the random in area $\|\vec{t}\| < T$ then the normed variable has a form

$$\left(\int_{\|\vec{t}\|<T} d\vec{t}\right)^{\frac{1}{2}} \left(\vec{u}(T)-\vec{u_0}\right)$$

and

$$\left(\int_{\|\vec{t}\|<T} d\vec{t}\right)^{\frac{1}{2}} \left(F_T\left(\vec{u}(T)\right)-F\left(\vec{u_0}\right)\right).$$

Further we consider a case where the restrictions are of the form

$$J=\left\{u : g(u)=\left(g_1(u),...,g_n(u)\right)\le 0\right\}.$$

Then the family of vectors

$$\boldsymbol{\eta}_T=\left(\int_{\|\vec{t}\|<T} d\vec{t}\right)^{\frac{1}{2}} \left(\vec{u}(T)-\vec{u_0}\right)$$

converges weakly to the random vector $\boldsymbol{\eta}_T$ which is the solution to the problem

$$\frac{1}{2}\vec{u}\,\boldsymbol{\Phi}\left(\vec{u_0}\right)\vec{u}+\varsigma\,\vec{u}\to\min;$$

$$\nabla g^T\left(\vec{u_0}\right)\vec{u}\le 0,$$

where ς is normal random vector.

The following problem consists in obtaining some theorems of the large deviations for a method of empirical means for the dependent observations. We will give some results. Our purpose consists in receiving the following estimates

$$\limsup_{n\to\infty}\frac{1}{n}\ln P\{|\min\{Ef(x),x\in X\}-\min\{F_n(x),x\in X\}|\ge\varepsilon\}\le-\inf\{I(z),z\in A_\varepsilon\}$$

and

$$\limsup_{n\to\infty}\frac{1}{n}\ln P\{|x_n-x^*|\ge\psi^{-1}(2\varepsilon)\}\le-\inf\{I(z),z\in A_\varepsilon\},$$

where $I(x)$ is the some positive function of some look on which we will stop later, and ψ is the some so-called improving function – monotonously not decreasing function, $\psi:[0,\infty)\to[0,\infty),\psi(0)=0$], such that it exists $\rho>0$, for which at all $x\in\mathcal{B}(x,f,\rho)$ we have

$$f(x)\ge f(x_f)+\psi\left(\|x-x_f\|\right). \qquad (2)$$

For the formulation of the main results we need a concept of hypermixing which is given in the monograph [5]. Let us recollect some facts from the functional analysis. For any $y\in Y$ the function $f(\circ,y)$ belongs to the space $C(X)$ of continuous real functions on X. We assume that for all $y\in Y$ we have $f(\circ,y)-Ef(\circ)\in K$, where K is some convex compact set from $C(X)$. Therefore for any $nF_n(\circ)-Ef(\circ)$ is a random element defined on the probability space $(\Omega,\mathcal{F},P)$ and with values in K.

Definition [5]. A random sequence $\{\xi_i,i\in\mathcal{N}\}$ is ergodic and stationary in the restricted sense is called a sequence with hypermixing if there exist a number $l_0\in\mathcal{N}\cup\{0\}$ and non-increasing functions $\alpha,\beta:\{l>l_0\}\to[1,+\infty)$ and $\gamma:\{l>l_0\}\to[0,1]$ which satisfy the next conditions

$$\lim_{l\to\infty}\alpha(l)=1,\quad \limsup_{l\to\infty} l(\beta(l)-1)<\infty,\quad \lim_{l\to\infty}\gamma(l)=0$$

and for which

$$\left\|\eta_1\ldots\eta_p\right\|_{L^1(P)}\le\prod_{j=1}^{p}\left\|\eta_j\right\|_{L^{\alpha(l)}(P)} \tag{3}$$

whenever $p\ge 2, l>l_0, \eta_1,\ldots,\eta_p$ are l-measurably separated functions. Here

$$\left\|\eta\right\|_{L^r(P)}=\left(\int_\Omega\left|\eta(\omega)\right|^r dP\right)^{1/r}$$

and

$$\left|\int_\Omega\left(\xi(\omega)-\int_\Omega\xi(\omega)dP\right)\eta(\omega)dP\right|\le\gamma(l)\left\|\xi\right\|_{L^{\beta(l)}(P)}\left\|\eta\right\|_{L^{\beta(l)}(P)}$$

for all $l>l_0$, $\xi,\eta\in L^1(P)$ are l-measurably separated. The following statements take place.

Theorem 2.6. [6] *At the hypothesis* (3) *of the hypermixing condition we have*

$$\limsup_{n\to\infty}\frac{1}{n}\ln P\left\{\left|\min\{Ef(x),x\in X\}-\min\{F_n(x),x\in X\}\right|\ge\varepsilon\right\}\le-\inf\{I(z),z\in A_\varepsilon\}\cdot$$

Assume that there exists a function ψ which meets a condition (2) *for $Ef(x)$ in the point x^*. Let x_n be a point of the minimum of the function $F_n(x)$ in the set $B(x^*,\rho)$. If ε is sufficiently small so that the condition for*

$$\psi\left(\left|x-x^*\right|\right)\le 2\varepsilon\ \Rightarrow\left|x-x^*\right|\le\rho$$

is fulfilled, then

$$\limsup_{n\to\infty}\frac{1}{n}\ln P\left\{\psi\left(\left|x_n-x^*\right|\right)\ge 2\varepsilon\right\}\le-\inf\{I(z),z\in A_\varepsilon\}.$$

Moreover, if ψ is convex and strictly increasing on $[0,\rho]$ *then*

$$\limsup_{n\to\infty}\frac{1}{n}\ln P\left\{\left|x_n-x^*\right|\ge\psi^{-1}(2\varepsilon)\right\}\le-\inf\{I(z),z\in A_\varepsilon\},$$

$$A_\varepsilon=\{z\in K:\|z\|\ge\varepsilon\}, K\subset C(X).$$

The proof is based on the following statement.

Theorem 2.7. [6] *Suppose that $\{\xi_i, i\in Z\}$ is a stationary in a strict sense ergodic random sequence satisfying the hypothesis* (3) *of the hypermixing condition, defined on a probability space $(\Omega,\mathcal{F},P)$ with values in a compact convex set $K\subset C(X)$. Then for any measure $Q\in M(X)$ there exists*

$$\Lambda(Q)=\lim_{n\to\infty}\frac{1}{n}\ln\left(\int_\Omega\exp\left\{\sum_{i=1}^{n}\int_X\xi_i(\omega)(x)Q(dx)\right\}dP\right)$$

and for any closed $A\subset K$

$$\limsup_{n\to\infty}\frac{1}{n}\ln\left(P\left\{\frac{1}{n}\sum_{i=1}^{n}\xi_i\in A\right\}\right)\le-\inf\{\Lambda^*(g),g\in A\},$$

where

$$\Lambda^*(g)=\sup\left\{\int_X g(x)Q(dx)-\Lambda(Q),Q\in M(X)\right\}$$

is the nonnegative, lower semicontinuous and convex function.

3. Summary

The article was considered by the method of empirical mean, which is one of the most effective methods for solving stochastic programming problems. It is closely related to the so-called

M-estimates, which are widely used in the theory of statistical estimation of stochastic processes. The results of the asymptotic behavior of the estimates obtained by empirical means were presented.

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