UDC 519.876.5+550/34]:623.421.4 DOI: 10.31673/2412-4338.2024.019903

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PARAMETRIC MATHEMATICAL MODEL OF SEISMOACOUSTIC MONITORING OF A SINGLE MORTAR EXPLOSION

Abstracts. *The article is devoted to constructing a mathematical model of a single mortar explosion signal for automated seismoacoustic monitoring systems to identify mortar weapons for remote reconnaissance. Structural analysis and identification of the dynamic parameters of such objects is an extremely important topic when monitoring them to classify the weapons used for remote reconnaissance. A new mathematical model for identifying a mortar explosion is proposed, which reflects the most significant aspects of the monitoring process, which includes both the process itself and the interference and background noise accompanying this process imposed on a natural research. The article presents a technique for identifying the main structural parameters, such as the leading natural frequencies and the quality factor of the structure at these frequencies. The work proposed a new, previously unused method for assessing the identification of a mortar explosion, within the framework of which an original mathematical model was proposed that solves these problems.*

The proposed model is a nonlinear regression problem. To find an approximate solution to such a problem, the authors use non-convex optimization methods, for example, to find local minima - Livenberg-Marquardt gradient methods, and to find a global minimum, the Monte Carlo method using specific sequences is effective. In some cases, it is possible to search for local extrema in the vicinity of given vectors of values of all parameters when, for nonlinearly entering parameters, there is only one root closest to the given value of the corresponding parameter.

As a signal model, a superposition of solutions of a second-order differential equation was chosen, which describes a superposition of oscillators that entered at different times, having their eigenfrequency and corresponding amplitudes.

The optimal estimation of the signal parameters consists in determining the vector of free parameters that minimize the value of the criterion of agreement between the model and the observed data. Such a model is supported by the fact that it gives good agreement in the case of modeling a linear system of oscillating objects and, thus, takes into account the oscillating nature of the observed data and its simplicity. Thus, the presented model displays each type of mortar firing shots into its n-dimensional vector of informative parameters, making classifying small arms possible. To evaluate the informative parameters of the proposed model of the automated seismo-acoustic monitoring system, the article solves the problem of nonlinear regression, considering them as the point of the criterion optimum in the n-dimensional space.

Key words: *mathematical model, sesmoacoustic monitoring, Monte Carlo method, nonlinear regression, vector of informative parameters, Livenberg-Marquardt method, explosion identification.*

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ПАРАМЕТРИЧНА МАТЕМАТИЧНА МОДЕЛЬ СЕЙСМОАКУСТИЧНОГО МОНІТОРИНГУ ОДИНОЧНОГО ВИБУХУ МІНОМЕТА

Анотація. *Стаття присвячена побудові математичної моделі сигналу одиночного вибуху міномета для автоматизованих систем сейсмоакустичного моніторингу ідентифікації мінометного озброєння для дистанційної розвідки. Структурний аналіз та ідентифікація динамічних параметрів таких об'єктів є надзвичайно важливою темою при їх моніторингу для класифікації зброї, яка використовується для дистанційної розвідки. Запропоновано нову математичну модель ідентифікації вибуху мінометa, яка відображає найбільш суттєві аспекти процесу моніторингу, який включає як сам процес, так і перешкоди та фоновий шум, що супроводжує цей процес, накладений на природне дослідження. У статті представлено методику ідентифікації основних структурних параметрів, таких як передні власні частоти та добротність конструкції на цих частотах. У роботі запропоновано новий, раніше не використовуваний метод оцінки ідентифікації вибуху мінометa, в рамках якого запропоновано оригінальну математичну модель, що вирішує ці проблеми.*

Запропонована модель є задачею нелінійної регресії. Для наближеного вирішення такої задачі автори використовують невипуклі методи оптимізації, наприклад, для знаходження локальних мінімумів - градієнтні методи Лівенберга-Марквардта, а для знаходження глобального мінімуму ефективний метод Монте-Карло з використанням конкретних послідовностей. У деяких випадках можливий пошук локальних екстремумів в околі заданих векторів значень усіх параметрів, коли для нелінійно вхідних параметрів існує лише один корінь, найближчий до заданого значення відповідного параметра.

В якості моделі сигналу обрано суперпозицію розв'язків диференціального рівняння другого порядку, яка описує суперпозицію осциляторів, що ввійшли в різний час, мають власну частоту та відповідні амплітуди.

Оптимальна оцінка параметрів сигналу полягає у визначенні вектора вільних параметрів, які мінімізують значення критерію узгодження моделі з даними спостереження. Така модель підтверджується тим фактом, що вона дає хорошу узгодженість у випадку моделювання лінійної системи коливальних об'єктів і, таким чином, враховує коливальний характер спостережуваних даних і їх простоту. Таким чином, представлена модель відображає кожен тип пострілів мінометів у своєму n-вимірному векторі інформативних параметрів, що дозволяє класифікувати стрілецьку зброю. Для оцінки інформативних параметрів запропонованої моделі автоматизованої системи сейсмоакустичного моніторингу в статті розв'язано задачу нелінійної регресії, розглядаючи їх як точку оптимуму критерію в n-вимірному просторі.

Ключові слова: *математична модель, сесмоакустичний моніторинг, метод Монте-Карло, нелінійна регресія, вектор інформативних параметрів, метод Лівенберга-Марквардта, ідентифікація вибуху.*

1. Introduction.

Creating monitoring systems for conducting remote surveillance has become a natural necessity. This paper discusses a method for constructing mathematical models of seismic-acoustic monitoring to conduct remote surveillance. It presents a mathematical model of seismic-acoustic monitoring for assessing single signals of mortar explosions to identify the weapon firing the shot. Seismoacoustic monitoring is different in that it has its specific frequency range. It covers the seismic and sound range. This article proposes a previously unused method for assessing the identification of a mortar explosion. Within the framework of this method, an original mathematical model that solves these problems is proposed.

In modern conditions, it is necessary to create automated monitoring systems for detecting and identifying sources of explosions to obtain information and make decisions about their nature. To solve these problems, it is necessary to create mathematical models and algorithms for detecting and identifying explosions. In the monitoring approach, an object is identified with a point in the multidimensional space of model free parameters. Thus, the identification process determines a vector of parameters in the feature space. This paper proposes a mathematical model for identifying the type of mortar by identifying an explosion when studying seismic recordings of mortar explosions. Object identification problems involve decision-making problems. When choosing a mathematical model, it is necessary to select the space of informative features in such a way as to reduce the likelihood of errors of two types. The choice of model is also determined by the possibility of its use in the flow of signals generated by a series of explosions. The method for constructing a mathematical model is presented in this work.

It is necessary to build a mathematical model of a mortar explosion, which would reflect the most significant moments of the monitoring process, including the process itself and the interference and noise background accompanying this process, imposed on a natural experiment. A priori knowledge of the interference of a random process will significantly weaken its influence on obtaining estimates of process parameters, which is perceived as a useful signal. This attenuation is achieved by optimizing processing procedures and considering priori statistics of the random interference process.

A new mathematical model for displaying a mortar explosion signal in the seismoacoustic frequency range and constructive algorithms for its implementation are proposed. The mathematical properties of the model are studied. Theoretical provisions are confirmed by calculations for preliminary processing and extraction of the mortar explosion signal from field observation data.

To estimate the informative parameters of the studied explosion signal of the mortar, a nonlinear regression problem is solved, considering them as the optimum point of the criterion in n-dimensional space.

2. Mathematical modeling of seismoacustic monitoring.

When estimating model parameters in seismoacoustic monitoring problems and the general case, we are faced with such a representation of the model of observed seismoacoustic fields $y(t,x)$ when observations are complicated by additive noise $n(t,x)$ (t - time, and x - spatial coordinates, the latter, depending on the organization of the observation system, can be one-dimensional, two-dimensional and three-dimensional, or take one single value). The actual model of the field formation process $M(\boldsymbol{\alpha},t,x)$ is determined by the vector $\boldsymbol{\alpha}$ of free parameters of the model, and the model itself is the researcher's hypothesis about the modeled process. Moreover, the vector of free parameters of the model $\boldsymbol{\alpha}$ contains the parameters included in the model, both linearly and nonlinearly. So the model:

$$y(t,x) = M(\boldsymbol{\alpha},t,x) + n(t,x) \tag{1}$$

Here, the free parameters of the model to be estimated are the elements of the vector $\boldsymbol{\alpha}$, $n(t,x)$ is the additive noise. In general, the dimensions of vectors also need to be determined.

To completely solve a problem means determining the vector of free parameters of model $M(\boldsymbol{\alpha},t,x)$ for the selected means of $\boldsymbol{\alpha}$. The norm of the noise $n(t,x)$ is estimated for the vector $\boldsymbol{\alpha}$ calculation. If it does not exceed the threshold set by the researcher, then a decision is made about the adequacy of the hypothetical model $M(\boldsymbol{\alpha},t,x)$ to the process $y(t,x)$. In the general case, the dimension of the vector-free parameters of the model must also be determined. In what follows, we will consider the vector of free parameters $\boldsymbol{\alpha} \in A$, which A is the set of all possible vector values $\boldsymbol{\alpha}$.

To solve problem (1), we can propose a selection method [1], i.e. we are looking for such values of the parameter vector $\boldsymbol{\alpha}$ that, for the selected scalar product in the Hilbert space, give the minimum deviation of the model $M(\boldsymbol{\alpha},t,x)$ from the observed data $y(t,x)$. The Hilbert space is chosen, first of all, due to the fact that we always consider the noise $n(t,x)$ as a random process for which statistical characteristics are specified, and it is possible to calculate covariances, which are the scalar product. So, we come to the need to solve the following problem:

$$\min_{\vec{\alpha}\in A}\left[\left(y(t,x)-M(\boldsymbol{\alpha},t,x),y-M(\boldsymbol{\alpha},t,x)\right)\right] \quad (2)$$

Here $\left(n(t,x),n(t,x)\right)$ is the square of the noise norm at point x, and ε is the square of the norm of the a priori expected discrepancy between the imperfect model and the natural process that is being modeled. The value ε is determined subjectively as the researcher's attitude to the quality of the model. This approach has been the subject of many works in the past [2]; it is still relevant in geophysics today [3].

In (2), we defined the relation to the optimal estimates of the model parameters, firstly, as a point in the parameter space that minimizes the selected criterion and, secondly, the estimate is accepted only in the case when the value of the criterion does not exceed a certain threshold chosen a priori. Otherwise, the model is rejected.

Since the right side of the criterion contains the norm of the noise component, the way of choosing the scalar product naturally arises. It must be selected so that the interference rate is minimal. This norm is the dispersion of the random process at the point t.

$$\left(n(t,x_1),n(t,x_2)\right)=E\left(n(t,x_1)n(t,x_2)\right) \quad (3)$$

Here E is the operator for calculating the mathematical expectation of a random process with zero mathematical expectation at the point t.

To obtain unconditionally optimal estimates of the vector $\boldsymbol{\alpha}$, it is possible to minimize (3) by enumerating of $\boldsymbol{\alpha}$. But, as a rule, other approaches are used, for example, gradient methods or the Monte Carlo method using special sequences [4]

Mathematical model of the mortar explosion signal.

As a signal model, we choose a superposition of solutions of a second-order differential equation, which describes a superposition of oscillators that entered at different times, having their eigenfrequency and corresponding amplitudes.

$$M(t,\boldsymbol{\lambda})=\sum_{i=0}^{I}\Phi(t-\lambda_{0+4i})\lambda_{1+4i}\left[e^{-\lambda_{2+4i}(t-\lambda_{0+4i})}\sin[\lambda_{3+4i}(t-\lambda_{0+4i})]\right];\boldsymbol{\lambda}=\lambda_{p+4i}\ {}_{i=\overline{0,I};\,p=\overline{0,4}}. \quad (3)$$

Here $\boldsymbol{\lambda}$ is the vector of free parameters of the model, I is the number of submodels participating in the superposition, p is the number of the corresponding submodel, and $\Phi(t)$ is the Heaviside unit function [5].

The optimal estimation of the signal parameters consists in determining the vector of free parameters that minimize the value of the criterion of agreement between the model and the observed data. Such a model is supported by the fact that it gives good agreement in the case of modeling a linear system of oscillating objects and, thus, takes into account the oscillating nature of the observed data and its simplicity. We chose a relatively simple case, and as a fitting criterion, the value of the norm L_2 (the integral of the squared deviation of the model from the observed data $y(t)$) or L_1 (the integral of the modulus of the model deviation from the observed data $y(t)$). In the first case, the criterion takes the form $F(\boldsymbol{\lambda})$:

$$F(\boldsymbol{\lambda})=\int_{\tau}^{\tau+T}\left[y(t)-M(t,\boldsymbol{\lambda})\right]^2 dt, \quad (4)$$

In the second case, the criterion takes the form $F(\boldsymbol{\lambda})$:

$$F(\boldsymbol{\lambda}) = \int_{\tau}^{\tau+T} \left| y(t) - M(t,\boldsymbol{\lambda}) \right| dt$$

And the optimal estimate of free parameters $\boldsymbol{\lambda}^*$ is a point in the parameter space minimizing (2):

$$F(\boldsymbol{\lambda}^*) = \min_{\boldsymbol{\lambda}\in\boldsymbol{\Lambda}} F(\boldsymbol{\lambda}).$$

Here $y(t)$ is the analytical approximation of the vector of values of the processed observed data presented in Fig. 1. Λ is the set of possible values of the vector $\boldsymbol{\lambda}$.

To find the minimum of the criterion, we need to calculate partial derivatives $\frac{\partial F(\boldsymbol{\lambda})}{\partial \lambda_k}$, $k = \overline{0,K}$ and, equating them to zero, create a system of equations that looks like this:

$$\frac{\partial F(\boldsymbol{\lambda})}{\partial \lambda_k} = \int_{\tau}^{\tau+T} \left[y(t) - M(t,\boldsymbol{\lambda}) \right] \frac{\partial M(t,\boldsymbol{\lambda})}{\partial \lambda_k} dt = 0, \quad k = \overline{0,K}. \quad (5)$$

The system of equations is reduced to the following form:

$$\int_{\tau}^{\tau+T} \left[y(t) \cdot \mathbf{D}\left(M(t,\boldsymbol{\lambda})\right) \right] dt = \int_{\tau}^{\tau+T} \left[M(t,\boldsymbol{\lambda}) \cdot \mathbf{D}\left(M(t,\boldsymbol{\lambda})\right) \right] dt. \quad (6)$$

Here $\mathbf{D}\left(M(t,\boldsymbol{\lambda})\right)$ is a vector composed of functions, each of which is the partial derivative of the model with respect to the corresponding component of the vector $\boldsymbol{\lambda}$. For model (3), this vector has the form:

$$\mathbf{D}\left(M(t,\boldsymbol{\lambda})\right) = \left\{ \frac{\partial M(t,\boldsymbol{\lambda})}{\partial \lambda_{p+i}} \right\}; \; p = \overline{0,3}; \; i = \overline{0,I}. \quad (6)$$

Here i is the ordinal number of the submodel in (1). The relevance of the variational approach in solving geophysical problems can be traced, for example, in [6].

Model (3) represents a nonlinear regression problem. To solve it concerning the parameters nonlinearly included in the model, it is necessary to find the minimum of criterion (4), which has a large number of local extrema that are close in the parameter space; you need to get to the global minimum point in the Monte Carlo method, so as not to reach a local minimum and obtain a suboptimal solution. Note the convergence in probability of the algorithm for finding the minimum criterion (for each sensor) using the Monte Carlo method.

The situation is much simpler for parameters linearly included in the model since the functional (4) (for fixed nonlinear parameters) is strictly concave, and the extremum point is unique.

To find an approximate solution to such a minimization problem, non-convex optimization methods can be used, for example, to find local minima - gradient methods or Newton's method, and to find a global minimum, the Monte Carlo method using particular sequences is effective. In some cases, it is possible to search for local extrema in the vicinity of given vectors of values of all parameters, when for nonlinearly entering parameters only one root is found, which is closest to the given value of the corresponding parameter.The search for the minimum is carried out according to the Levenberg-Marquard algorithm [7,8] for a priori randomly selected point in the space of free parameters of the model (1).

3. Analysis of results.

Let's move on to analyzing the quality of the optimal model. Models with sixteen free parameters are presented. The optimized procedure for estimating the dynamic parameters of a mortar explosion recording with characteristics in the seismic frequency range is illustrated by processing field observations obtained when recording mortar explosions at the test site.

The model's quality is evaluated based on the criterion's value at the global minimum point. In Figure 6, crucial characteristics of the object are depicted, particularly the system's quality factor at natural frequencies. The figure illustrates the rates of energy dissipation or accumulation at natural frequencies for the 16-parametric model, considering three natural frequencies. A significant observation emerges: the quality factor varies notably across different frequencies (reflected in the time constant of the exponential). The initial point of each curve on the ordinate axis provides an estimation of the amplitude for each harmonic.

The frequencies and logarithmic decrements of the studied object hold significant physical meaning. The latter are particularly vital as they offer insights into the system's quality factor, its capacity to store and preserve energy from external disturbances over time. A high-quality factor (low decrement) at certain frequencies in the model signifies the object's heightened susceptibility to external disturbances at those frequencies. For instance, dynamic variations in decrement towards a decrease indicate the object's vulnerability to weak external influences, potentially leading to its deterioration. Regrettably, the Monte Carlo method employed in the article only converges to a solution probabilistically. It necessitates a substantial number of calculation cycles to ensure confidence in the accuracy of the result, especially challenging when dealing with high-dimensional models. Or you need to have good a priori ideas about the expected result. The following are the optimal parameters for the signal.

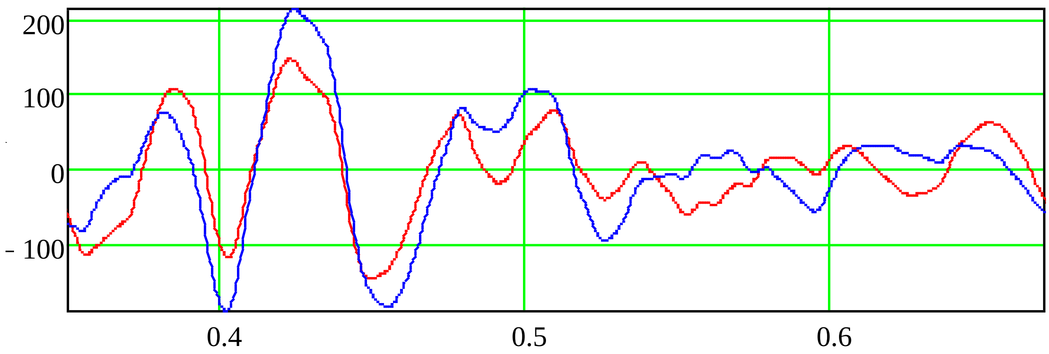

Fig.1. A fragment of a recording of a mortar explosion signal against background noise (blue line). A model that approximates this signal, the free parameters of which are obtained by assessing the optimal parameters of criterion 16 (red line). The abscissa axis represents time in seconds, and the ordinate axis represents the amplitude of the oscillation speed in relative units.

Transposed vector of amplitudes of natural frequencies:

$$\mathbf{S}^T = \{1{,}102 \quad 0{,}638 \quad 0{,}324\}$$

Transposed vector of decrements at natural frequencies:

$$\boldsymbol{\mu}^T = \{0{,}076 \quad 0{,}068 \quad 0{,}938\}$$

Transposed natural frequency vector:

$$\omega^T = \{0{,}291 \ \ 2{,}436 \ \ 15{,}831\} \text{ Гц}$$

The discrepancy between the resulting model and the observed data in Fig. 1 is 7% in metric. L_2 .

4. Conclusions.

A mathematical model for identifying a mortar explosion is considered, reflecting the most significant aspects of the monitoring process, including the process itself and the interference and noise background accompanying this process imposed on the investigation. The model is a nonlinear regression problem for which nonlinear optimization solution approaches are used.

Mathematical models of automated systems of seismoacoustic monitoring are used to model fields of mechanical elastic waves [9-14]. This paper presents such a model for identifying mortar weapons for remote reconnaissance. We can see that the informative parameters of the signal model characterize the above-described process with a high degree of sufficiency. We can conclude that this parametric model maps the process into the feature space and characterizes the object that fires the shots. Thus, the presented model (14) displays each type of mortar firing shots into its n-dimensional vector of informative parameters, making classifying small arms possible. Thus an effective analysis method is proposed for estimating the parameters of mortar explosion signals, and non-traditional model of the natural background against which signals of mortar explosions are recorded is proposed.

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