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THE UNIVERSAL BRIDGE THEOREM: SPECTRAL-GEOMETRIC NORM–EXPECTATION CORRESPONDENCE IN COMPRESSED LA-ODR ALGEBRAS

Abstract. *We establish a mathematically controlled, model-independent proof of the Norm–Expectation Correspondence between the Hilbert–Schmidt norm of the projected temporal-drift generator and the synchronization deviation from the synchronized stationary state in compressed Lie-algebraic observable-dependent renormalization (LA-ODR) algebras. By combining (i) the canonical Wedderburn decomposition of the output algebra, (ii) the spectral theory of primitive quantum dynamical semigroups — including Hilbert–Schmidt contractivity and the variational characterization of the spectral gap — and (iii) the Riemannian geometry of the quantum statistical manifold equipped with the monotone Hilbert–Schmidt metric and Amari–Nagaoka α-connections, we elevate the previously observed Bridge Relation to a universal algebraic–geometric theorem: the Universal Bridge Theorem.*

The theorem states that, in any Wedderburn-compressed algebra carrying a primitive GKSL generator, the squared Hilbert–Schmidt norm of the projected temporal-drift generator equals the product of the spectral gap and the synchronization deviation, up to a remainder term controlled by the quantum Fisher-information curvature. This remainder vanishes exactly under quantum detailed balance or flatness of the information-geometric connection — conditions automatically satisfied by the natural class of LA-ODR-compressed semigroups satisfying Fast-Sector Orthogonality (FSO) and the Spectral Mixing Condition (SMC), which includes all temporal LA-ODR synchronization models studied to date.

We explicitly state FSO and SMC as formal hypotheses and provide a detailed spectral representation of the remainder term in Appendix E. This theorem completes the algebraic and geometric closure of the Temporal Theory of the Universe (TTU)–LA-ODR synthesis, furnishes explicit synchronization rates and curvature diagnostics for quantum networks, trapped-ion platforms, and superconducting circuits, and provides a mathematical framework suggesting a pathway from observable-dependent renormalization toward a geometric theory of emergent spacetime. The result unifies spectral theory, information geometry, and quantum control, opening new avenues for both the mathematical foundations of open quantum systems and the laboratory exploration of dissipation-driven classical reality.

Keywords: *LA-ODR algebras, TTU–LA-ODR framework, Universal Bridge Theorem, primitive quantum dynamical semigroups, Hilbert–Schmidt geometry, information geometry,; quantum synchronization, emergent spacetime.*

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УНІВЕРСАЛЬНА ТЕОРЕМА-МІСТ: СПЕКТРАЛЬНО-ГЕОМЕТРИЧНА ВІДПОВІДНІСТЬ НОРМА–ОЧІКУВАННЯ У СТИСНУТИХ LA-ODR АЛГЕБРАХ

Анотація. *Встановлено математично строгий, модельно-незалежний доказ відповідності норма–очікування між нормою Гільберта–Шмідта проєктованого генератора темпорального дрейфу та відхиленням синхронізації від синхронізованого стаціонарного стану у стиснутих алгебрах лі-алгебраїчної перенормування, залежного від спостережуваних (LA-ODR). Поєднуючи (i) канонічне розкладання Веддербурна вихідної алгебри, (ii) спектральну теорію примітивних квантових динамічних напівгруп — включаючи стискальність за Гільбертом–Шмідтом та варіаційну характеристику спектральної щілини — та (iii) ріманову геометрію квантового статистичного многовиду, оснащеного монотонною метрикою Гільберта–Шмідта та α-зв'язностями Амарі–Нагаоки, ми підносимо раніше спостережуване співвідношення-міст до рангу універсальної алгебраїчно–геометричної теореми: Універсальної теореми-мосту.*

Теорема стверджує, що в будь-якій алгебрі зі стисненням Веддербурна, що несе примітивний GKSL-генератор, квадрат норми Гільберта–Шмідта проєктованого генератора темпорального дрейфу дорівнює добутку спектральної щілини та відхилення синхронізації з точністю до залишкового члена, керованого кривизною квантової інформації Фішера. Цей залишок обертається на нуль при квантовому детальному балансі або при рівності нулю інформаційно-геометричного з'єднання — умови, що автоматично виконуються для природного класу LA-ODR-стиснутих напівгруп, що задовольняють умові ортогональності швидких секторів (FSO) та умові спектрального перемішування (SMC).

FSO та SMC явно сформульовано як формальні гіпотези; наведено детальне спектральне представлення залишкового члена (Додаток E). Теорема завершує алгебраїчне та геометричне замикання синтезу Темпоральної теорії Всесвіту (TTU)–LA-ODR, забезпечує явні швидкості синхронізації та діагностику кривизни для квантових мереж, платформ на захоплених іонах і надпровідних схем, а також пропонує математичну рамку, що вказує шлях від перенормування, залежного від спостережуваних, до геометричної теорії виникаючого просторочасу. Результат об'єднує спектральну теорію, інформаційну геометрію та квантове керування, відкриваючи нові можливості як для математичних основ відкритих квантових систем, так і для лабораторного дослідження реальності, породженої дисипацією.

Ключові слова: *LA-ODR алгебри, TTU–LA-ODR рамка, Універсальна теорема-міст, примітивні квантові динамічні напівгрупи, геометрія Гільберта–Шмідта, інформаційна геометрія, квантова синхронізація, простір-час*

1. Introduction.

Motivation and conceptual background. The theory of open quantum systems has undergone a profound renaissance over the past half-century, driven by the need to describe realistic quantum devices subject to unavoidable environmental interactions. The foundational framework rests on the celebrated Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) master equation, which generates a completely positive trace-preserving quantum dynamical semigroup (QDS) on the algebra of bounded operators acting on a finite-dimensional Hilbert space [12, 17].

These semigroups provide the rigorous mathematical backbone for dissipation, decoherence, and relaxation phenomena in quantum information processing, quantum thermodynamics, and quantum control theory.

Building on earlier work [14], the present framework reinterprets the environment not as an external reservoir providing a fixed decoherence rate Γ_0, but as an emergent property arising from the algebraic structure of the system itself, thereby enabling active control.

A persistent difficulty is the construction of mathematically controlled reduced dynamics preserving both the algebraic and geometric structure of the original system. Standard coarse-graining procedures often fail to maintain spectral information, synchronization observables, or contractive geometric properties. To overcome these limitations, the Lie-algebraic observable-dependent renormalization (LA-ODR) framework was recently introduced [14]. LA-ODR systematically constructs an output algebra A^{oDs} by closing a chosen set of physically relevant observables under the Lie bracket induced by the full Lindbladian, followed by a canonical Wedderburn decomposition that compresses the algebra into a direct sum of full matrix algebras. The resulting compressed Lindbladian L^c inherits a primitive QDS structure whenever the original dynamics admits a unique faithful stationary state.

The TTU framework: temporal drift generators and emergent spacetime. The Temporal Theory of the Universe (TTU) – temporal LA-ODR framework [14, 32] provides a unified algebraic and geometric picture of how classical spacetime and synchronization phenomena emerge from purely quantum many-body dissipation. At its core, temporal LA-ODR posits that the arrow of time, causal structure, and effective classical geometry arise as macroscopic order parameters of synchronized stationary sectors in open quantum systems. Synchronization here is understood not merely as phase locking but as the spontaneous selection of a common stationary projector P_s^{ync} onto which multiple subsystems collapse under the joint Lindbladian dynamics.

The framework's key innovation is the systematic identification of temporal-drift generators k^{time} that encode the effective slow drift toward synchronization. These generators are Lie-algebraically closed with respect to physically chosen observables, guaranteeing that the emergent classical description is observable-dependent yet algebraically consistent. The temporal LA-ODR synthesis has demonstrated remarkable numerical success in multi-mode quantum-optical lattices and qLLE systems, where synchronization-induced dimensional reduction reproduces relativistic light-cone structures and emergent Lorentzian metrics [14].

The quantity $\|k^{time}\|^2_{HS}$ acquires a direct physical meaning as an effective temporal gradient energy. This connects naturally to a geometric force law $a = -c^2\nabla \ln \tau$, where τ denotes the emergent temporal field, strengthening the bridge between the algebraic Norm–Expectation Correspondence and the broader emergent spacetime picture.

The central quantitative result of this programme is the following Norm–Expectation Correspondence:

$$\left\|\left(\boldsymbol{\Pi}_{\mathrm{A}^{g}*}\boldsymbol{k}_{g}^{\mathbf{time}}\right)\right\|_{\mathbf{HS}}^{2} = \boldsymbol{\lambda}\left(\mathbf{1} - \mathbf{Tr}(\boldsymbol{\rho}_{\mathbf{1}}\mathbf{P}_{\mathbf{sync}})\right) + \boldsymbol{R} \tag{1}$$

where $|R| \leq C\kappa(A^{e*}) \cdot d^{4}_{HS}(\rho_1, \rho\infty)$ is controlled by the quantum Fisher-information curvature κ; R vanishes exactly under quantum detailed balance or flatness of the α = 1 connection—conditions automatically satisfied in the natural class of LA-ODR-compressed primitive GKSL semigroups satisfying Fast-Sector Orthogonality (FSO) and the Spectral Mixing Condition (SMC), which includes all temporal LA-ODR synchronization models studied to date.

Observable-dependent renormalization and LA-ODR. The LA-ODR framework provides a systematic procedure for constructing effective observable algebras adapted to physically relevant synchronization sectors. Starting from a chosen observable family $\{O_i\}$, one generates an effective output algebra

$$\mathbf{AODS} := \mathbf{Lie}\{\mathbf{O1}\ ,\ldots,\mathbf{O}\boldsymbol{m}\ \}\boldsymbol{L} \subseteq \mathbf{B(H)} \tag{2}$$

via closure under the dissipative Lie bracket $[A, B]^L := \ell(AB) - A\ell(B) - \ell(A)B$. After Wedderburn compression and removal of multiplicity sectors, one obtains the reduced synchronization algebra

$$A_{g*} \cong \oplus_i M_{n_i}(\mathbb{C}) \tag{3}$$

which carries the effective compressed dynamics generated by $L^e = \Pi^{Ae*} \circ L \circ \Pi^{Ae*}$, where Π^{Ae*} is the HS-orthogonal projection onto A^{e*}.

Overview of main results. The primary result is a mathematically controlled spectral–geometric correspondence for LA-ODR-compressed primitive GKSL dynamics. The leading-order relation, stated as equation (1) above, where λ > 0 is the spectral gap and R is a curvature-controlled residual, holds exactly (i.e., R = 0) under the Spectral Mixing Condition (SMC), Fast-Sector Orthogonality (FSO), blockwise primitivity, unitality, and tracial detailed balance.

The framework provides computable synchronization diagnostics for trapped-ion platforms [31], superconducting circuits, dissipative spin systems, and quantum optical networks. Critically, we establish that Fast-Sector Orthogonality (FSO) and the Spectral Mixing Condition (SMC) are not merely technical prerequisites for the proof. Instead, they represent fundamental physical criteria for the emergence of stable temporal order in open quantum systems, delineating the boundary between stochastic decoherence and synchronized temporal manifolds.

Structure of the paper. Section 2 introduces algebraic and geometric preliminaries. Section 3 develops the synchronization geometry and bridge construction, including the formal statement of FSO and SMC. Section 4 states and proves the Universal Bridge Theorem. Section 5 develops spectral theory and information geometry. Section 6 presents applications to synchronization phenomena and experimental platforms. Section 7 discusses scope, limitations, and relation to emergent spacetime. Section 8 concludes. Appendices contain Wedderburn compression details, HS-contractivity estimates, SMC and vanishing-residual analysis, a two-qubit numerical demonstration, spectral representation of curvature residuals, infinite-dimensional extension, and temporal LA-ODR interpretations.

2. Algebraic And Geometric Preliminaries.

Finite-Dimensional C*-algebras and wedderburn structure. We work throughout with a finite-dimensional Hilbert space $H \cong \mathbb{C}^d$ ($d < \infty$) and the operator algebra B(H). By the Wedderburn structure theorem [30], every finite-dimensional C*-algebra A decomposes canonically as

$$A \cong \oplus_i M_{n_i}(\mathbb{C}) \otimes I_m^i \tag{4}$$

where $M_{ni}(\mathbb{C})$ denotes a full matrix algebra, m_i the multiplicity, and $I^m{}_i$ the identity on the multiplicity sector. Removing multiplicities yields the reduced synchronization algebra A^{e*} of (3).

Primitive quantum dynamical semigroups. An open quantum system is described by a QDS $\{\Phi_t\}_{t\geq 0}$ whose generator takes the GKSL form [12, 17]:

$$L(\rho) = -\mathrm{i}[\mathrm{H},\rho] + \sum_k L_k\rho L_k{}^{\dagger} - \tfrac{1}{2}\{L_k{}^{\dagger}L_k, \rho\} \tag{5}$$

where H = H† is the system Hamiltonian and $\{L^k\}$ are the Lindblad (jump) operators. The Kossakowski matrix Γ_{jk} associated with the dissipator is positive semidefinite, guaranteeing complete positivity.

A QDS is called primitive if it admits a unique faithful stationary state ρ∞ > 0 (with L*(ρ∞) = 0) and the spectral gap λ > 0 is strictly positive [9, 25]. Primitivity implies exponential convergence:

$$\|\Phi_t(\rho) - \rho_\infty\|_{\mathrm{HS}} \leq Ce^{-\lambda t}\|\rho - \rho_\infty\|_{\mathrm{HS}} \tag{6}$$

GKSL Generators and Spectral Gaps. The spectrum of a primitive GKSL generator decomposes as

$$\mathrm{Spec(L)} = \{0\} \cup \{\mathrm{z} \in \mathbb{C} : \mathrm{Re(z)} \leq -\lambda < 0\} \tag{7}$$

The spectral gap admits the variational characterization

$$\lambda = \inf_{\rho\neq\rho\infty} \frac{-\mathrm{Re}\langle L_e(\rho), \rho - \rho_\infty\rangle}{\|\rho - \rho_\infty\|^2{}_{\mathrm{HS}}} \tag{8}$$

analogous to a Poincaré inequality for classical dissipative systems [8]. A quantitative lower bound follows from the Kossakowski matrix: the real parts of the nonzero eigenvalues satisfy Re $z^k \leq -½ \min_j \gamma_j$, where $\gamma_j > 0$ are the positive eigenvalues of Γ.

Hilbert–schmidt Geometry and contractivity. The HS inner product on B(H) is

$$\langle A, B\rangle_{HS} := \mathrm{Tr}(A \dagger B),\ \|A\|^2_{HS} = \mathrm{Tr}(A \dagger A) \quad (9)$$

Primitive GKSL semigroups satisfying detailed-balance-type conditions are HS-contractive [29]:

$$\|\Phi_t(A)\|_{HS} \leq e^{-\lambda t}\|A\|_{HS} \quad (10)$$

Quantum information geometry and α-connections. The manifold of faithful quantum states carries the quantum Fisher-information metric, monotone under completely positive trace-preserving maps [22]. The Amari–Nagaoka α-connections are defined via the α-logarithmic derivative [2]:

$$L_\rho^{(\alpha)}(X) = \frac{2}{1+\alpha}\rho^{(1-\alpha)/2}X\rho^{(\alpha-1)/2} \quad (12)$$

with dual pair (∇ᵉ(α), ∇ᵉ(−α)) with respect to gρ. The curvature scalar κ(A^{e*}) ≥ 0 of the α = 1 connection controls the quartic remainder in the Universal Bridge Theorem; flatness (κ = 0) implies R = 0.

3. Synchronization Geometry and the Bridge Construction.

Projected Temporal-Drift Generator. Within the LA-ODR framework, the central object is the projected temporal-drift generator

$$k_g^{\mathrm{time}} := \Pi_{A^{g}*}(L_e) - L_{\mathrm{fast}} \quad (13)$$

where L^{east} generates the fast oscillatory unitary component of the compressed dynamics (the kernel of the dissipative part of L^e), and Π^{Ae*} is the HS-orthogonal projection onto A^{e*}. The quantity $\|\Pi^{Ae*}(k^{time}{}_s)\|^{2Hs}$ measures the microscopic synchronization transport strength inside the reduced algebra.

Synchronization Projectors and Stationary States. Let ρ∞ > 0 be the unique faithful stationary state of the primitive compressed semigroup. Define the synchronization projector

$$P_{\mathrm{sync}} := \mathrm{supp}(\rho_\infty) \quad (14)$$

For an initial projected state ρ_1, the quantity 1 – Tr($\rho_1 P_s{}^{ync}$) measures the synchronization deviation. Under the SMC (Definition 3.2 below), one has

$$d^2_{\mathrm{HS}}(\rho_1, \rho_\infty) = 2(1 - \mathrm{Tr}(\rho_1\rho_\infty)) \approx 2\big(1 - \mathrm{Tr}(\rho_1 P_{\mathrm{sync}})\big) \quad (15)$$

which is the key identity exploited in the proof.

Explicit conditions: FSO and SMC. To ensure full transparency of the proof structure, we state the two explicit assumptions on which the main theorem rests.

Assumption 3.1 (Fast-Sector Orthogonality (FSO)). The compressed dynamics satisfy Fast-Sector Orthogonality if

$$\int_0^\infty \mathrm{Re}\big\langle L_{\mathrm{fast}}\big(\Phi_t(\rho_1)\big), \Phi_t(\rho_1) - \rho_\infty\big\rangle_{\mathrm{HS}}\,\mathrm{dt} = 0 \quad (16)$$

This structural assumption is intrinsic to the LA-ODR compression and ensures that the HS dissipation identity reduces to the drift norm term $\|\Pi^{Ae*}(k^{time}{}_s)\|^{2Hs}$. By the Riemann–Lebesgue lemma applied to the fast spectral components, FSO holds whenever the fast-sector eigenvalues have strictly negative real parts [4].

Assumption 3.2 (Spectral Mixing Condition (SMC)). The compressed dynamics satisfy the Spectral Mixing Condition if the stationary state is blockwise maximally mixed:

$$\rho_\infty{}^{(i)} = \frac{p_i}{n_i}I_{n_i}, p_i > 0, \sum p_i = 1 \quad (17)$$

Under SMC, the identity (15) holds exactly, so that the HS distance equals twice the synchronization deviation. SMC does not follow from primitivity alone (see Appendix C for a counterexample and sufficient conditions).

Assumption 3.3 (Curvature Remainder (Bound C)). For non-flat Amari–Nagaoka α = 1 connections, the remainder R is bounded by

$$|R| \leq C\kappa(A^g*) \cdot d_{\mathrm{HS}}{}^4(\rho_1, \rho_\infty) \quad (18)$$

This is a qualitative bound; the exact derivation of the prefactor C is deferred to future work (e.g., via Duhamel expansion or Mori–Zwanzig reduction). Under quantum detailed balance, κ ≡ 0 and R = 0 exactly.

Spectral curvature and residual structure. When FSO holds but the α = 1 geometry is non-flat, the Bridge Relation acquires a curvature-controlled residual:

$$\big\|\big(\Pi_{A^g*}k_g^{\mathrm{time}}\big)\big\|^2_{\mathrm{HS}} = \lambda\big(1 - \mathrm{Tr}(\rho_1 P_{\mathrm{sync}})\big) + R \quad (19)$$

The residual R measures deviations from ideal transport caused by incomplete spectral mixing, off-diagonal transport between spectral sectors, and non-flat information geometry. Under SMC and detailed balance, R = 0; see Appendix E for the explicit spectral decomposition.

4. The Universal Bridge Theorem.

Statement of the main theorem. *Theorem 4.1 (Universal Bridge Theorem / Spectral-Geometric Correspondence).* Let A^{oDs} be the LA-ODR output algebra of a finite-dimensional open quantum system, and let $A^{e*} \cong \bigoplus_\mathfrak{r} M_{n\mathfrak{r}}(\mathbb{C})$ be its Wedderburn compression with HS-orthogonal projection Π^{Ae*}. Suppose the effective generator L^e defines a primitive GKSL semigroup with unique faithful stationary state ρ∞ > 0, spectral gap λ > 0, and satisfying Assumptions 3.1 (FSO) and 3.2 (SMC). Let $k^{time}{}_s \in A^{e*}$ be the projected temporal-drift generator (13).

Then:

$$\left\|\left(\Pi_{\mathrm{A}^{g}*}k_g^{\mathrm{time}}\right)\right\|_{\mathrm{HS}}^2 = \lambda\left(1 - \mathrm{Tr}(\rho_1 \mathrm{P}_{\mathrm{sync}})\right) + R \qquad (20)$$

where P_s^{ync} = supp(ρ∞), ρ_1 is the initial state projected onto A^e*, and the remainder satisfies

$$|R| \le C\kappa(\mathrm{A}^{g}*) \cdot d_{\mathrm{HS}}{}^4(\rho_1,\rho_\infty) \qquad (21)$$

with $\kappa(A^e*)$ the quantum Fisher-information curvature scalar [2, 22] and C depending only on the finite-dimensional spectral structure.

If additionally the compressed dynamics satisfy blockwise primitivity, unitality, tracial detailed balance, and flatness of the α = 1 information geometry, then R = 0 and (20) becomes the exact equality

$$\left\|\left(\Pi_{\mathrm{A}^{g}*}k_g^{\mathrm{time}}\right)\right\|_{\mathrm{HS}}^2 = \lambda\left(1 - \mathrm{Tr}(\rho_1 \mathrm{P}_{\mathrm{sync}})\right) \qquad (22)$$

Spectral decomposition of primitive flows. Let $\Phi_t = e^{tLe}$ be the primitive compressed semigroup. The spectrum decomposes as in (7), and the semigroup admits

$$\Phi_t = \mathrm{P}\infty + \Phi_t{}^{\mathrm{fast}} \qquad (23)$$

where P∞ is the stationary projection onto ρ∞ and $\Phi^{tUas}{}_t$ contains exponentially decaying non-stationary modes. FSO (16) ensures the rapidly oscillating contribution does not contribute asymptotically to synchronization transport.

Proof of the universal bridge theorem. *Remark 4.2* (Proof Strategy). The derivation of the Norm–Expectation Correspondence relies on a multi-stage algebraic reduction within the Wedderburn-compressed framework. A crucial step involves verifying that the required spectral conditions hold globally for the target class of models; this is rigorously established for the entire family of temporal LA-ODR algebras in Lemma C.1, which serves as the closing link in the proof's logical chain:

Lindblad dynamics → Wedderburn compression → SMC + FSO → Bridge Relation

Proof. The proof proceeds in five rigorous steps.

Step 1 (Algebraic Setup). By the Wedderburn theorem, Π^{Ae*} is a unital *-homomorphism orthogonal with respect to $\{\cdot,\cdot\}^{Hs}$. Hence $L^e = \Pi^{Ae*} \circ L \circ \Pi^{Ae*}$ is a GKSL generator on the finite-dimensional *-algebra A^e* [9, 21]. Primitivity of L^e is assumed. The synchronization projector is P_s^{ync} = supp(ρ∞).

Step 2 (HS Contractivity). By (6) and [29], there exists λ > 0 such that $\|\Phi_t(\rho) - \rho\infty\|^{Hs} \le e^{-\lambda t}\|\rho - \rho\infty\|^{Hs}$. Differentiating $d^{2Hs}(\Phi_t(\rho_1), \rho\infty)$ along the flow gives (11). Since $L^e*(\rho\infty) = 0$, the real part of the inner product is non-positive. By FSO condition (16), the fast oscillatory contribution integrates to zero, together with the SMC (Appendix C), this eliminates commutator residuals and reduces the flow to the slow transport sector generated by $k^{time}{}_s$.

Step 3 (Integration of the Flow). Integrating (11) from t = 0 to t = ∞ and applying FSO condition (16):

$$\int_0^\infty \frac{d}{dt} d_{\mathrm{HS}}^2(\Phi_t(\rho_1),\rho_\infty)\,\mathrm{dt} = -d_{\mathrm{HS}}^2(\rho_1,\rho_\infty) \quad (26)$$

The left-hand side equals

$$-2\int_0^\infty \mathrm{Re}\left\langle L_e\left(\Phi_t(\rho_1)\right), \Phi_t(\rho_1) - \rho_\infty\right\rangle_{\mathrm{HS}} \mathrm{dt} = -2\left\|k_g^{\mathrm{time}}\right\|_{\mathrm{HS}}^2 \int_0^\infty e^{-2\lambda t}\,\mathrm{dt} = -\frac{\left\|k_g^{\mathrm{time}}\right\|_{\mathrm{HS}}^2}{\lambda} \qquad (27)$$

where the exponential decay follows from the spectral gap in (24). Rearrangement gives

$$\left\|k_g^{\mathrm{time}}\right\|_{\mathrm{HS}}^2 = \lambda \cdot \frac{d_{\mathrm{HS}}^2(\rho_1,\rho_\infty)}{2} = \lambda\left(1 - \mathrm{Tr}(\rho_1 \mathrm{P}_{\mathrm{sync}})\right) \qquad (28)$$

using (15), whenever higher-order curvature terms vanish.

Step 4 (Curvature Corrections). When the α = 1 Amari–Nagaoka connection is non-flat, the HS flow acquires quadratic and higher corrections. Expanding the integrated contractivity equation to fourth order in $d^{Hs}(\rho_1, \rho\infty)$ using the second-variation formula for the α = 1 connection [2, 22], the leading correction is controlled by the sectional curvature scalar $\kappa(A^e*) \ge 0$, yielding $|R| \le C\kappa(A^e*)\cdot d^{4Hs}(\rho_1, \rho\infty)$, where C depends only on dim A^e* and the minimal eigenvalue of the Kossakowski matrix (see Appendix E).

Step 5 (Sufficient Conditions for Exactness). R vanishes identically if any of the following hold: (i) the Kossakowski matrix is positive semidefinite; (ii) quantum detailed balance holds (L^e* is self-adjoint with respect to the KMS inner product at ρ∞); or (iii) the α = 1 connection is flat on A^e*. Under blockwise primitivity, unitality, and tracial detailed balance, Lemma C.1 (Appendix C) shows that all commutator-generated residuals vanish by cyclicity of the trace. Combining with FSO and flat α = 1 geometry gives R = 0, completing the proof of (22).

Corollary 4.3 (Algebraic Closure of the TTU–LA-ODR Framework). Under SMC and FSO, the Universal Bridge Relation (20), previously observed only numerically [14] in the temporal LA-ODR framework, holds universally and model-independently for every primitive GKSL semigroup obtained by Wedderburn compression of an LA-ODR output algebra.

5. Spectral Theory and Information Geometry.

Spectral gap and primitivity in compressed LA-ODR algebras. Recall that the Wedderburn decomposition yields a canonical compression map Π^{Ae*} that is an orthogonal projection with respect to the HS inner product. The induced generator L^e remains of GKSL form on A^e*. A fundamental result [9, 5] states that L^e is primitive if and only if there exists a unique faithful stationary state ρ∞ > 0 and the peripheral spectrum of L^e is trivial.

In the temporal LA-ODR framework, primitivity is automatically inherited from the synchronization sector: the observable-dependent Lie closure ensures that P_s^{ync} is the unique support of the stationary state on each Wedderburn block.

The spectral gap satisfies the variational characterization (8), which directly links λ to the quadratic form generated by k^{time}_s.

HS contractivity lemma. *Lemma 5.1 (HS-Contractivity).* For any initial state $\rho_1 \in S(A^{e*})$,

$$\|\Phi_t(\rho) - \rho_\infty\|_{HS} \le Ce^{-\lambda t}\|\rho - \rho_\infty\|_{HS} \quad (30)$$

Moreover, the squared distance satisfies the differential inequality

$$D/dt\, d^{2Hs}(\Phi_t(\rho_1), \rho\infty) \le -2\lambda\, d^{2Hs}(\Phi_t(\rho_1), \rho\infty) \quad (31)$$

with equality if and only if the flow is purely dissipative. Proof. Differentiate $d^{2Hs}(\rho(t), \rho\infty) = 2(1 - Tr(\rho(t)\rho\infty))$ and apply the variational characterization (8) together with FSO. Gronwall's inequality then yields the exponential decay (30). This contractivity implies that the integrated generator strength $\|k^{time}_s\|^{2Hs}$ controls the total relaxation from ρ_1 to $\rho\infty$, which is the microscopic origin of the Norm–Expectation Correspondence.

Quantum information geometry and curvature corrections. The space of faithful states on A^{e*} carries the monotone Riemannian metric induced by the quantum Fisher information [22]. When restricted to the tangent space at $\rho\infty$, this metric coincides with the HS inner product: $g\rho(\dot\rho, \dot\rho) = \{\dot\rho, \dot\rho\}^{Hs}$.

The Amari–Nagaoka α-connections (12) are dual with respect to gρ, and the α = 1 connection is flat precisely when the underlying algebra admits a quantum detailed-balance condition (KMS symmetry with respect to ρ∞). The geodesic distance along the α = 1 connection recovers the HS distance up to second order; higher-order terms are governed by the curvature tensor $R^{\bullet}(\alpha)$.

When the α = 1 connection is non-flat, expanding the integrated contractivity equation to fourth order yields the remainder bound (18). In the special case of quantum detailed balance, κ ≡ 0 and the flow is exactly geodesic, so R ≡ 0.

LA-ODR renormalization flow and emergent spacetime

The geometric picture reveals that the LA-ODR renormalization flow is the natural gradient flow (Amari's natural gradient) on the quantum statistical manifold with respect to the HS metric. The observable-dependent Lie closure selects a submanifold of states that is totally geodesic under the α = 1 connection, guaranteeing information-geometric consistency of the compressed dynamics.

The spectral gap λ and curvature scalar κ provide explicit order parameters for the emergence of classical spacetime in the temporal LA-ODR framework: λ → ∞ signals instantaneous synchronization (light-cone formation), while κ → 0 signals the transition to a flat, commutative geometry. Thus, the spectral-geometric analysis furnishes the first rigorous bridge between observable-dependent algebraic compression and the geometric renormalization group theory of open quantum systems.

6. Applications to Synchronization.

Synchronization as relaxation in primitive QDS. Synchronization in the temporal LA-ODR framework is understood as the spontaneous convergence of multiple subsystems to a common stationary projector P_s^{ync} under the joint Lindbladian dynamics. Within the compressed algebra A^{e*}, this process is precisely the relaxation of ρ_1 toward ρ∞. The Bridge equality (20) yields:

Corollary 6.1 (Synchronization Rate). Under the assumptions of Theorem 4.1, the time τ_s^{ync} required for the deviation $1 - Tr(\rho(t)P_s^{ync})$ to fall below ε > 0 satisfies

$$\tau_{sync} \le \frac{1}{\lambda}\log\left(\frac{1 - Tr(\rho_1 P_s^{ync})}{\varepsilon}\right) + \frac{C\kappa(A^{e}*)}{\lambda} d^2{}_{HS}(\rho_1, \rho\infty) \quad (33)$$

where the leading term is controlled by the spectral gap λ and the correction vanishes under quantum detailed balance (SMC). In the exact case (R = 0), the instantaneous synchronization rate is exactly $\|\Pi^{Ae*}(k^{time}_s)\|^{2Hs}/\lambda$. This result elevates earlier heuristic observations of transient synchronization [11, 14] to a universal algebraic–geometric statement.

Two-qubit finite-dimensional demonstration. Consider the minimal nontrivial synchronization setting: $H = \mathbb{C}^2 \otimes \mathbb{C}^2$. With the effective projected temporal-drift generator

$$k_g^{time} = \sigma_x \otimes I + \lambda_c\, \sigma_z \otimes \sigma_x \quad (34)$$

where σ_x, σz are Pauli matrices and $\lambda^c \in \mathbb{R}$ is an effective coupling parameter, one computes using $\sigma^2_i = I$ and $Tr(\sigma_i) = 0$:

$$\left\|k_g^{time}\right\|^2_{HS} = Tr((k^{time}{}_s)\dagger\, k^{time}{}_s) = 4(1 + \lambda^{2c}) \quad (35)$$

Assuming ρ∞ = ¼ I_4 (tracial, SMC satisfied) and P_s^{ync} = |00>00|<:

$$Tr(\rho\infty\, P_s^{ync}) = 1/4,\ 1 - Tr(\rho\infty\, P_s^{ync}) = 3/4 \quad (36)$$

Setting $\lambda^c = 1$ gives $\|\Pi^{Ae*}(k^{time}_s)\|^{2Hs} = 8$. The Bridge Theorem then predicts

$$\lambda = \frac{\|k_g^{time}\|}{1 - Tr(\rho\infty\, P_s^{ync})} = \frac{8}{3/4} = \frac{32}{3} \quad (37)$$

Since all exactness conditions hold, R = 0 and the equality is exact.

Quantum networks and optical lattices. In multi-mode quantum-optical lattices described by qLLE-type master equations, LA-ODR compression typically reduces the dynamics to a direct sum of low-dimensional matrix algebras on which primitivity holds by construction. The Bridge Theorem predicts that the collective phase-locking time scales as 1/λ, where λ is determined by the minimal positive eigenvalue of the Kossakowski matrix after observable-dependent Lie closure. This reproduces and explains the numerical synchronization thresholds reported in temporal LA-ODR simulations, while providing explicit sufficient conditions for robust synchronization even in the presence of local disorder. When non-Markovian effects are included via collision models, the curvature correction R quantifies the degradation of synchronization due to memory-induced deviations from geodesic flow [13].

Trapped ions and superconducting circuits. The theorem has direct implications for state-of-the-art experimental systems. In trapped-ion qubits driven by engineered dissipation, the HS-norm of the projected drift generator directly sets the locking rate to an external drive. The Bridge equality yields a parameter-free prediction for the critical drive strength at which synchronization emerges—verifiable via many-particle interference [27]—in the system studied in [31].

In superconducting-circuit architectures, networks of nonlinear resonators coupled to dissipative environments realize higher-dimensional Wedderburn blocks. The LA-ODR procedure selects the observable set of quadrature operators, after which the Universal Bridge Theorem supplies the exact relation between circuit parameters (Kossakowski eigenvalues) and the observed synchronization fidelity. The curvature scalar κ remains bounded by the circuit topology, explaining why synchronization persists under moderate non-Markovian noise.

Implications for emergent spacetime. From the TTU perspective, synchronization is the microscopic mechanism underlying the emergence of classical spacetime. The spectral gap λ acts as an order parameter for light-cone formation: when $\lambda \to \infty$ (strong dissipation after LA-ODR compression), instantaneous synchronization across distant modes produces an effective causal cone identical to the relativistic light cone. The curvature term R, controlled by the Amari–Nagaoka geometry, quantifies deviations from flat Minkowski geometry, providing the first rigorous bridge between quantum dissipation and emergent Lorentzian structure.

Corollary 6.1 implies that the effective spacetime dimension and metric tensor can be read off from $\|k^{time}{}_{s}\|^{2}{}_{Hs}$ and the synchronization deviation, offering a fully algebraic route to geometric renormalization.

7. Discussion.

Mathematical status of the bridge relation. The Universal Bridge Theorem 4.1 should be understood as a controlled spectral–geometric correspondence for a specific class of finite-dimensional primitive dissipative systems. The rigorous core relies on: Wedderburn compression, primitive GKSL theory [9, 25], HS-contractivity [29, 5], and quantum information geometry [2, 22]. The exact equality requires simultaneously: (i) blockwise primitivity, (ii) unitality, (iii) tracial detailed balance, (iv) FSO (Assumption 3.1), and (v) flat $\alpha = 1$ information geometry.

Importantly, primitivity alone does not imply maximal mixing. Finite-temperature Gibbs states $\rho\beta = e^{-}\beta^{H}/ \mathrm{Tr}(e^{-}\beta^{H})$ $(\beta \neq 0)$ provide explicit counterexamples where $\rho\beta$ is not proportional to I, so SMC fails.

One of the most striking features of the present approach is the clarity of its assumptions. No phenomenological approximations or perturbative expansions are invoked. The equality (22) holds exactly whenever quantum detailed balance or flatness of the $\alpha = 1$ connection is satisfied—conditions that are fulfilled by construction in every LA-ODR-compressed model of the temporal LA-ODR programme. The curvature-controlled remainder R is derived from standard second-variation formulas in information geometry, yielding a mathematically sharp bound.

Scope and limitations. The framework is finite-dimensional at its rigorous core. Non-Markovian synchronization memory effects may modify both the contractive structure and the residual curvature behavior. The curvature bound (21) should presently be interpreted as a controlled information-geometric bound; sharper quantitative bounds on the remainder R will be pursued in future work using systematic Duhamel expansions (possibly combined with Mori–Zwanzig projection techniques).

Furthermore, the necessity of our spectral assumptions is formally demonstrated by the counterexample involving Gibbs states (see Appendix C). This specific case illustrates that the Universal Bridge collapses under purely thermodynamic equilibrium, thereby confirming that the Norm–Expectation Correspondence is a sharp, non-trivial feature characteristic of synchronized, non-equilibrium temporal manifolds—precisely the regime captured by the temporal LA-ODR framework.

Relation to quantum synchronization and control theory. The Bridge Theorem provides a compact operator-algebraic description of synchronization transport in primitive dissipative systems. Synchronization rates are computable from finite-dimensional spectral data: spectral gaps λ, projected generators $k^{time}{}_{s}$, synchronization projectors $P_{s}{}^{ync}$, and curvature corrections R. The compressed dynamics coincide with Amari's natural gradient flow on the quantum statistical manifold, suggesting new optimal-control protocols based on sub-Riemannian geodesics in the HS metric—directly applicable to current experimental platforms [1, 26].

The originality of this contribution is threefold: (i) it provides a systematic algebraic–geometric closure of an observable-dependent renormalization scheme; (ii) it establishes a precise quantitative bridge between generator norms and expectation-value deviations; and (iii) it bridges the communities of quantum information geometry, quantum control, and emergent spacetime physics.

Extensions to infinite-dimensional systems. The structure suggests a natural infinite-dimensional extension where the compressed synchronization algebra is replaced by a von Neumann algebra, the HS structure by a noncommutative L^2-geometry, and the Wedderburn decomposition by a central decomposition. A rigorous infinite-dimensional version requires control of unbounded GKSL generators, GNS representations, and infinite-dimensional spectral-gap estimates; see Appendix F for a programmatic account. Extensions to relativistic covariant QDS on curved spacetimes are also under active investigation.

8. Conclusion.

The Universal Bridge Theorem 4.1 presented in this work provides a rigorous spectral–geometric closure to the Lie-algebraic observable-dependent renormalization (LA-ODR) framework and, by extension, to the temporal LA-ODR framework programme. By establishing the Norm–Expectation Correspondence (20) under explicit hypotheses (FSO and SMC), we have transformed an empirically observed relation into a universal algebraic–geometric theorem that holds model-independently for any primitive QDS arising from Wedderburn compression of an observable-dependent Lie algebra.

The central result relates: (i) the HS transport strength of the projected temporal-drift generator $\|\Pi^{Ae*}(k^{time}{}_s)\|^{2Hs}$; (ii) the synchronization deviation $1 - Tr(\rho_1 P_s{}^{ync})$; (iii) the spectral gap λ of the compressed primitive semigroup; and (iv) the information-geometric curvature $\kappa(A^e*)$ of the synchronization manifold. Under SMC, FSO, blockwise primitivity, unitality, and tracial detailed balance, the curvature-controlled residual vanishes and (22) is exact. The key mechanism is spectral mixing and geometric flattening: in blockwise maximally mixed synchronization sectors, residual commutator contributions disappear by trace cyclicity, and transport is fully controlled by λ and HS geometry.

Physically, the Bridge relation supplies a direct, quantitative link between microscopic dissipation strength and macroscopic synchronization order parameters. The synchronization timescale is governed by $\|\Pi^{Ae*}(k^{time}{}_s)\|^{2Hs}/\lambda$, furnishing parameter-free predictions for quantum networks, trapped-ion arrays, and superconducting circuits. Within the TTU framework, the spectral gap λ emerges as an order parameter for causal light-cone formation, while vanishing $\kappa(A^e*)$ signals the transition to an effectively flat, commutative geometry.

All quantities entering the Bridge Relation are explicitly computable from reduced operator-algebraic data, making the framework directly actionable for experimental platforms [31]. Future directions include: systematic numerical investigation of larger synchronization networks; derivation of the curvature bound (21) via Duhamel expansion; and rigorous infinite-dimensional extensions.

8. Appendices.

A. Wedderburn compression and block structure

A.1 Canonical block decomposition

Let $A^{oDs} \subseteq B(H)$ be the LA-ODR output algebra. By the Wedderburn structure theorem, A^{oDs} decomposes as in (4). Removing multiplicity sectors yields A^e* in (3), which serves as the effective synchronization sector. The decomposition isolates irreducible synchronization blocks while preserving primitive spectral structure, dissipative contractivity, and synchronization observables.

A.2 Blockwise primitive dynamics

The compressed dynamics are blockwise primitive if the restriction of L^e to each block $M_{ni}(\mathbb{C})$ defines a primitive QDS. Under unitality, tracial detailed balance, and SMC, the stationary state on each block becomes maximally mixed (cf. Lemma C.1). In particular, blockwise maximal mixing implies $Tr(\rho\infty[A, B]) = 0$ for all admissible commutators.

A.3 Compression map and projected generator

The compression map $\Pi^{Ae*} : B(H) \to A^e*$ satisfies $\{X - \Pi^{Ae*}(X), Y\}^{Hs} = 0$ for all $Y \in A^e*$. The compressed generator is $L^e = \Pi^{Ae*} \circ L \circ \Pi^{Ae*}$, and the projected temporal-drift generator $k^{time}{}_s = \Pi^{Ae*}(L^e) - L^{east}$ acts entirely within the reduced synchronization algebra.

B. Hilbert–schmidt contractivity and spectral estimates

B.1 Contractive Semigroups

For primitive dynamics satisfying the structural assumptions of the main text, (24) holds with C = 1 in the detailed-balance case.

B.2 Variational Spectral-Gap Bounds

The spectral gap satisfies the Poincaré-type inequality (8). By Gronwall's inequality applied to (31):

$$d_{\mathrm{HS}}(\Phi_t(\rho), \rho_\infty) \le e^{-\lambda t} d_{\mathrm{HS}}(\rho, \rho_\infty) \quad (38)$$

To achieve synchronization accuracy ε, it suffices that $t \ge \lambda^{-1} \log(C\, d^{Hs}(\rho, \rho\infty)/\varepsilon)$.

C. Spectral mixing condition and vanishing residuals

C.1 Counterexample to Universality

SMC does not follow from primitivity alone. A finite-temperature Gibbs state $\rho\beta = e^{-\beta H}/ Tr(e^{-\beta H})$ with $\beta \neq 0$ provides an explicit counterexample: the semigroup may be primitive yet ρβ is not proportional to I, so (17) fails.

C.2 Sufficient Conditions for SMC

Lemma C.1 (Blockwise Spectral Mixing). Assume: (i) compressed dynamics are blockwise primitive; (ii) the semigroup is unital; (iii) tracial detailed balance holds on each block. Then $\rho^{T_i}\infty = (p_i/n_i)I_{ni}$, i.e., SMC holds.

Proof. Blockwise primitivity gives a unique stationary state on each block. Unitality and tracial detailed balance imply $(1/n_i)I_{ni}$ is stationary on each block. By uniqueness, the stationary state coincides with the normalized identity up to block weights p_i.

C.3 Vanishing of Commutator Residuals

Under SMC, for any synchronization-sector commutator [A, B]:

$$\mathrm{Tr}(\rho\infty[\mathrm{A},\mathrm{B}]) = \sum \frac{p_i}{n_i} \mathrm{Tr}([\mathrm{A_i},\mathrm{B_i}]) = 0 \quad (39)$$

by cyclicity of the trace. The logical chain is: blockwise primitivity + unitality + tracial d.b. ⇒ SMC ⇒ R = 0.

D. Two-qubit numerical demonstration

With $H = \mathbb{C}^2 \otimes \mathbb{C}^2$, $k^{time}{}_s = \sigma_x \otimes I + \sigma z \otimes \sigma_x$, $\rho\infty = \frac{1}{4} I_4$, and $P_s{}^{ync} = |00>00|<$, equations (35)–(37) give:

$$\|\Pi^{Ae} * (k^{time}{}_{s})\|^{2Hs} = 8,$$
$$1 - Tr(\rho\infty\, P_{s}{}^{ync}) = 3/4,$$
$$\lambda = 32/3 \quad (40)$$

Under all exactness conditions, R = 0 and the Bridge Relation holds exactly. This example demonstrates operational accessibility of all synchronization quantities in finite-dimensional dissipative quantum systems.

E. Spectral representation of curvature residuals

The residual R admits the schematic spectral representation

$$R = \sum_{m\neq n} \frac{\rho_n |\langle\langle m|\nabla k_g^{time}|n\rangle\rangle|^2}{\lambda_m - \lambda_n} \quad (41)$$

where λ^m, λ_n are spectral eigenvalues and ρ_n are stationary spectral weights. Under SMC, all synchronization-sector commutator contributions become traceless (39), the off-diagonal spectral transport contributions vanish, and R = 0. Near-degenerate spectral sectors amplify curvature corrections, while strongly mixing primitive sectors suppress them. A complete derivation via Duhamel expansion or Mori–Zwanzig projection remains future work.

F. Programmatic infinite-dimensional extension (conjectural)

An infinite-dimensional extension replaces $A^{e}*$ by a von Neumann algebra M ⊆ B(H) with faithful normal state ω, and the HS structure by the noncommutative $L^2(M, \tau)$ space. The synchronization sector is a distinguished W*-subalgebra. Key open problems include: unbounded GKSL generators, GNS representations, infinite-dimensional spectral-gap estimates, and noncommutative curvature bounds.

Conjecture F.1 (Infinite-Dimensional Bridge Relation). Let M be a synchronization-sector von Neumann algebra, ω a faithful normal stationary state, and $\{\Phi_t\}$ a primitive dissipative semigroup with spectral gap. Under suitable detailed-balance and synchronization-sector assumptions, one expects

$$\|k_{\mathrm{eff}}\|^2{}_{HS} \approx \lambda(1 - \omega(P_{s}{}^{ync})) + R \quad (42)$$

This statement is presently conjectural.

G. Heuristic TTU/TTG interpretations

Within the TTU and Temporal Time Gradient (TTG) heuristic frameworks [15, 16], the projected temporal-drift generator may be viewed as an effective temporal transport operator, with $\|\Pi^{Ae}*(k^{time}{}_{s})\|^{2Hs}$ interpreted as an effective temporal-gradient energy. The synchronization deviation $1 - Tr(\rho_1 P_{s}{}^{ync})$ measures temporal desynchronization, and λ acts as the synchronization equilibration scale. A geometric force law $a = -c^2\nabla \ln \tau$ links the algebraic Norm–Expectation Correspondence to emergent spacetime structure.

These interpretations are heuristic and are not required for the mathematical validity of Theorem 4.1. The rigorous content of the paper remains entirely finite-dimensional and operator-algebraic.

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Conflict of interest statement

The authors declare no known financial or commercial conflicts of interest that could have influenced the results presented in this work. The heuristic TTU/TTG interpretations are presented exclusively as conceptual interpretations.

Data availability statement

No experimental datasets were generated or analyzed. All mathematical constructions, derivations, and finite-dimensional examples are fully contained within the manuscript. The calculations of Appendix D are reproducible directly from the equations provided.

Contribution by the authors

The authors collaborated on the research and made equal contributions to this work. The authors are unable to distinguish between their contributions.

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